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A FORTRAN FOR SOLVING SYSTEMS OF COUPLED SECOND-ORDER DIFFERENT--ETC(U)
AUG 76 J M FORBES, H B GARRETT F19628-76-C-0059

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**A FORTRAN PROGRAM FOR SOLVING SYSTEMS OF
COUPLED SECOND-ORDER DIFFERENTIAL EQUATIONS
WITH TWO-POINT BOUNDARY CONDITIONS**

By

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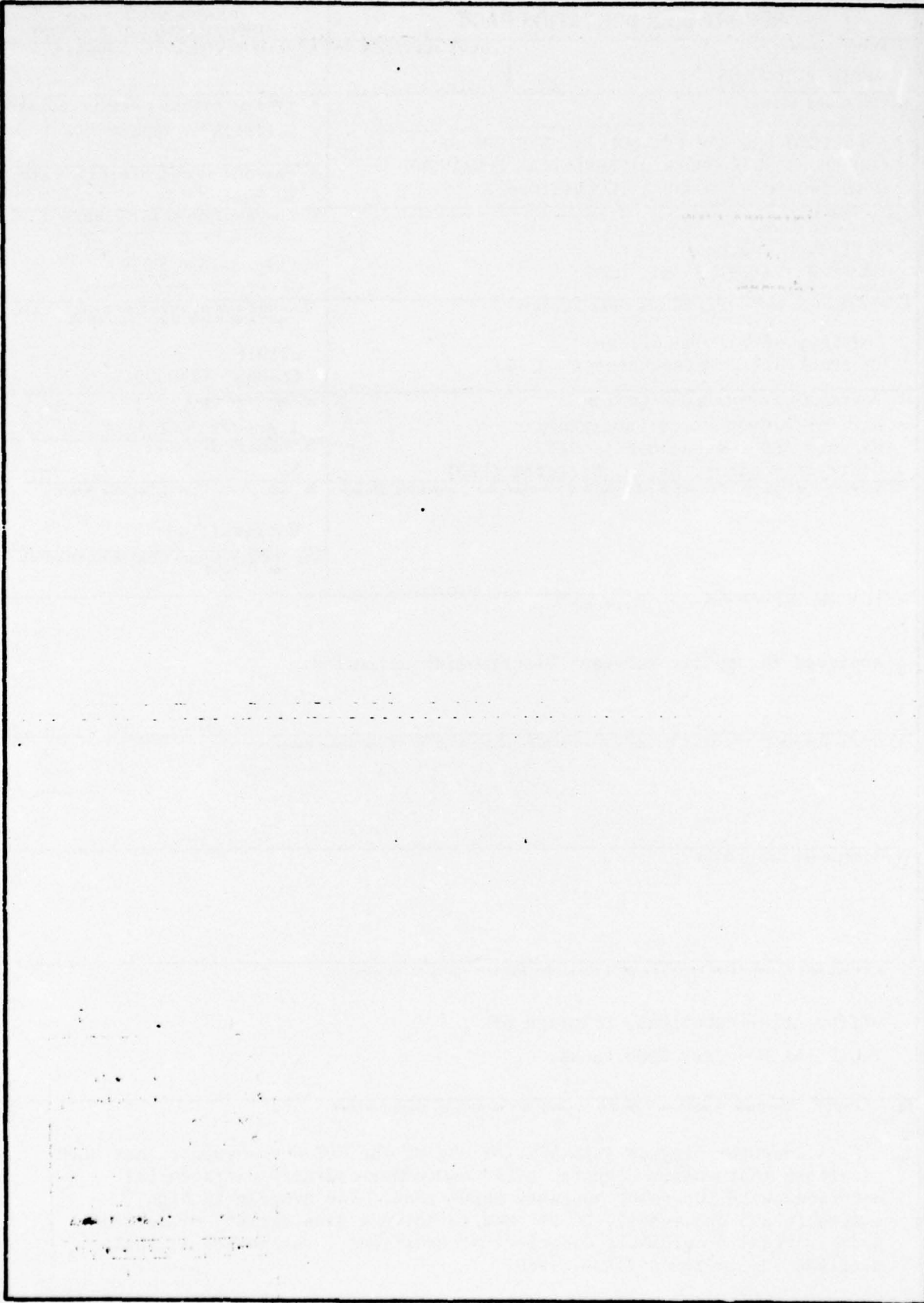
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TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION	1
METHOD OF SOLUTION	1
PROGRAM	4
EXAMPLE	5
REFERENCES	7
APPENDIX I	8
APPENDIX II	21
APPENDIX III	27

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A FORTRAN PROGRAM FOR SOLVING SYSTEMS OF COUPLED
SECOND-ORDER DIFFERENTIAL EQUATIONS WITH TWO-POINT
BOUNDARY CONDITIONS

INTRODUCTION

Systems of second-order ordinary differential equations with two-point boundary conditions are invariably encountered in problems of applied physics. Lindzen and Kuo (1969) have published an algorithm for solving such systems by the method of Gaussian Elimination. The method was found to be of particular value in the solution of various thermospheric tidal problems currently under investigation at Boston College and the Air Force Geophysics Laboratory. The following is a description of a computer program developed for the CDC 6600 computer which uses this technique. With the addition of a suitable user-supplied subroutine, the program can solve a wide variety of second-order ordinary and partial differential equations.

METHOD OF SOLUTION

A system of N coupled, second-order ordinary differential equations can be written in the following form:

$$\bar{C}'(x) \frac{d^2}{dx^2} \bar{F}(x) + \bar{A}'(x) \frac{d}{dx} \bar{F}(x) + \bar{B}'(x) \bar{F}(x) = \bar{R}'(x)$$

Where \bar{A}' , \bar{B}' , and \bar{C}' are $N \times N$ matrices and \bar{F} and \bar{R}' are N -dimensional vectors. To solve this system of equations numerically, we first change to finite differences. Letting $x_n = n\delta x$ ($n=1,2,\dots$):

$$\frac{d}{dx} F(x_n) \approx \frac{F_{n+1} - F_{n-1}}{2 \delta x}$$

and

$$\frac{d^2}{dx^2} F(x_n) \approx \frac{F_{n+1} - 2 F_n + F_{n-1}}{\delta x^2}$$

the new finite difference equations become:

$$1) \quad \bar{A}_n \bar{F}_{n-1} + \bar{B}_n \bar{F}_n + \bar{C}_n \bar{F}_{n+1} = \bar{R}_n$$

where

$$\bar{A}_n = \frac{\bar{C}'(x_n)}{(\delta x)^2} - \frac{\bar{A}'(x_n)}{2 \delta x}$$

$$\bar{B}_n = \frac{-2 \bar{C}'(x_n)}{(\delta x)^2} + \bar{B}'(x_n)$$

$$\bar{C}_n = \frac{\bar{C}'(x_n)}{(\delta x)^2} + \frac{\bar{A}'(x_n)}{2 \delta x}$$

$$\bar{R}_n = \bar{R}'(x_n)$$

Boundary conditions at x_1 and x_N are assumed to be of the form:

$$2) \quad \bar{C}'(x_1) \frac{d}{dx} \bar{F}|_{x_1} + \bar{A}'(x_1) \bar{F}(x_1) = \bar{R}'(x_1)$$

$$3) \quad \bar{C}'(x_N) \frac{d}{dx} \bar{F}|_{x_N} + \bar{A}'(x_N) \bar{F}(x_N) = \bar{R}'(x_N)$$

where x_1 is the value of x at the lower boundary and x_N is the value at the upper boundary. In finite difference form, these are:

$$4) \quad \bar{A}_1 \bar{F}_1 + \bar{B}_1 \bar{F}_2 = \bar{R}_1$$

$$5) \quad \bar{A}_N \bar{F}_{N-1} + \bar{B}_N \bar{F}_N = \bar{R}_N$$

$$\bar{A}_1 = \bar{A}'(x_1) - \frac{\bar{C}'(x_1)}{\delta x}$$

$$\bar{B}_1 = \frac{\bar{C}'(x_1)}{\delta x}$$

$$\bar{R}_1 = \bar{R}'(x_1)$$

$$\bar{A}_N = \frac{-\bar{C}'(x_N)}{\delta x}$$

$$\bar{B}_N = \frac{\bar{C}'(x_N)}{\delta x} + A'(x_N)$$

$$\bar{R}_N = \bar{R}'(x_N)$$

The solution (Richtmyer, 1957) is as follows. Assume:

$$6) \quad \bar{F}_n = \bar{\alpha}_n \bar{F}_{n+1} + \bar{\beta}_n$$

where $\bar{\alpha}_n$ and $\bar{\beta}_n$ are to be determined. Then:

$$7) \quad \bar{F}_{n-1} = \bar{\alpha}_{n-1} \bar{F}_n + \bar{\beta}_{n-1}$$

Equation 7, when substituted into 1, yields:

$$(\bar{A}_n \bar{\alpha}_{n-1} + \bar{B}_n) \bar{F}_n + (\bar{C}_n) \bar{F}_{n+1} = (\bar{R}_n - \bar{A}_n \bar{\beta}_{n-1})$$

Comparing to Equation 6 yields:

$$8) \quad \bar{\alpha}_n = -(\bar{A}_n \bar{\alpha}_{n-1} + \bar{B}_n)^{-1} \bar{C}_n$$

$$9) \quad \bar{\beta}_n = (\bar{A}_n \bar{\alpha}_{n-1} + \bar{B}_n)^{-1} (\bar{R}_n - \bar{A}_n \bar{\beta}_{n-1})$$

At the lower boundary, using Equation 6:

$$\bar{F}_1 = \bar{\alpha}_1 \bar{F}_2 + \bar{\beta}_1$$

Therefore:

$$10) \quad \bar{\alpha}_1 = -(\bar{A}_1)^{-1} \bar{B}_1$$

$$11) \quad \bar{\beta}_1 = (\bar{A}_1)^{-1} \bar{R}_1$$

Likewise, Equations 5 and 7 can be solved to give:

$$12) \quad \bar{F}_N = (\bar{A}_N \bar{\alpha}_{N-1} + \bar{B}_N)^{-1} (\bar{R}_N - \bar{A}_N \bar{\beta}_{N-1})$$

We now have the means of solving the equations.

To review, we first calculate the finite difference form of the equations. $\bar{\alpha}_1$ and $\bar{\beta}_1$ are computed. Then we generate the other $\bar{\alpha}_n$ and $\bar{\beta}_n$, using Equations 8 and 9, through $\bar{\alpha}_N$ and $\bar{\beta}_N$ (note: $\bar{\alpha}_N$ is not needed and $\bar{F}_N = \bar{\beta}_N$). The α and β , with Equation 6, generate the \bar{F}_n , completing the solution.

PROGRAM

The computer formulation is straightforward (see Appendix I for listing). The main program, TIDE, calls subroutines ABCR, ABCRN, ALPBET, and SOL which perform the steps outlined in the previous description.

Subroutine ABCR is to be supplied by the user. This program calculates matrices \bar{A}' , \bar{B}' , and \bar{C}' and vector \bar{R}' for a given value of x . By proper manipulation of this subroutine, a variety of ordinary and partial differential equations can be solved.

Subroutine ABCRN calculates the matrices \bar{A}_n , \bar{B}_n , and \bar{C}_n and vector \bar{R}_n in the finite difference forms given in Equations 1, 4, and 5. Note that the matrices A , B , and C and vector R are reused. As the typical dimensions of such matrices are 40 x 40 or greater for our applications, this is a necessary process.

Subroutine ALPBET calculates $\bar{\alpha}_n$ and $\bar{\beta}_n$ from Equations 8, 9, 10, and 11. They are stored on TAPE 4. TAPE 3 is necessary as a work tape for otherwise twice as many matrices would be required. If the matrix dimensions permit, this tape could be deleted and replaced by storage matrices, with substantial savings in computer time.

Subroutine SOL uses Equation 7 to obtain the final solutions which appear in FNC. The output, in FNC, is printed, but it could be stored on a tape, plotted, or punched.

EXAMPLE

Appendices II and III contain the results of a sample calculation for 3 ordinary, second-order differential equations. Appendix II is the exact solution while Appendix III contains the numerical solution.

The example was defined as follows:

$$\bar{A}'(x) = \begin{vmatrix} 0 & 0 & i x^3 \\ 0 & x^2 & 0 \\ 0 & 0 & x^2 \end{vmatrix} \quad \bar{B}'(x) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x^2 \end{vmatrix}$$

$$\bar{C}'(x) = \begin{vmatrix} 2 & 0 & 0 \\ i x^3 & x^2 & 0 \\ 0 & 0 & 0 \end{vmatrix} \quad \bar{R}'(x) = \begin{vmatrix} (4-3 x^3) + i(4 x^6) \\ i(8 x^3 + 3 x^4) \\ (4 x^5 + x^6) + (3 x^2 + 3 x^3)i \end{vmatrix}$$

$i = \sqrt{-1}$

The boundary conditions are, for $x = 0$ and $x = T$:

$$\bar{A}'(0) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 0 \end{vmatrix}$$

$$\bar{B}'(0) = 0$$

$$\bar{C}'(0) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\bar{R}'(0) = \begin{vmatrix} 0 \\ 3i \\ 0 \end{vmatrix}$$

For $x = T$:

$$\bar{A}'(T) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & T^2 & 0 \end{vmatrix}$$

$$\bar{B}'(T) = 0$$

$$\bar{C}'(T) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix}$$

$$\bar{R}'(T) = \begin{vmatrix} T^2 \\ T^4 + 3 T i \\ i(6 T^2 + T^5) \end{vmatrix}$$

The solutions to this system of equations are:

$$\bar{F}(x) = \begin{vmatrix} x^2 \\ i x^3 \\ x^4 + (3 x) i \end{vmatrix}$$

REFERENCES

Lindzen, R.S., and H.L. Kuo, "A Numerical Method for the Numerical
Integration of a Large Class of Ordinary and Partial Differential
Equations", Monthly Weather Review, Vol. 97, No. 10, 732-734, 1969.

Richtmyer, R.D., Difference Methods for Initial-Value Problems, Interscience
Press, New York, 1957.

APPENDIX I

MAIN PROGRAM TIDE

PURPOSE

NUMERICALLY INTEGRATES EQUATION $C(D^2/DX^2)F + A(D/DX)F + BF = R$.
WHERE A,B,C ARE MATRICES AND F,R ARE VECTORS. A,B,C,P
ARE GIVEN BY SUBROUTINE ABCR. F IS TO BE COMPUTED

USAGE

PROVIDE INPUT SUBROUTINE ABCR. OUTPUT AT STATEMENTS 10
AND 11.

DESCRIPTION OF PARAMETERS

A - INPUT MATRIX
B - INPUT MATRIX
C - INPUT MATRIX
R - INPUT VECTOR
BET - STORAGE VECTOR
FNC1 - STORAGE VECTOR
FNC - OUTPUT VECTOR
DETERM - DETERMINANT VALUE FROM MATRIX INVERSION. ILL-
CONDITIONED PROBLEM WILL GIVE 0.
IR - NUMBER OF ROWS
IC - NUMBER OF COLUMNS
S1 - INITIAL VALUE OF X
SN - FINAL VALUE OF X
N - NUMBER OF STEPS

REMARKS

USER SHOULD CHANGE DIMENSIONS AS APPROPRIATE. IN THIS
CASE WHERE THE NUMBER 3 APPEARS. OUTPUT IS AT STATEMENTS
10 AND 11.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

ABCR - SETS UP PROBLEM
ABCRN - CONVERTS USER'S PROBLEM TO FINITE DIFFERENCE FORM.
ALPBET - COMPUTES INTERMEDIATE SOLUTIONS ALPHA AND BETA
GABS - SYSTEM FUNCTION
SOL - COMPUTES FINAL SOLUTIONS F(X)

METHOD

SEE LINDZEN AND KUD, MONTHLY WEATHER REVIEW, VOL. 97, NO.
10, OCT 1969, 732-734.

PROGRAM TIDE(INPUT,OUTPUT,TAPE3=513,TAPE4)

CHANGE DIMENSTONS

COMPLEX A(3,3),B(3,3),C(3,3),P(3)
COMPLEX BET(3),FNC(3),FNC1(3),DETERM
IR=3
IC=3

INTEGRATE FROM:

S1=0

```

C
C      10:
C
C      SN=2
C
C      IN STEPS OF:
C
C      DX=.01
C
C      NUMBER OF STEPS WILL BE:
C
C      N=(SN-S1)/DX+1.001
100 FORMAT(5(1X, E13.6))
      DO 7 I=1,N
      X=DX*FLOAT(I-1)+S1
C
C      SET UP MATRICES
C
C      CALL ABCR(A,B,C,R,I,N,X,IR,IC)
C
C      COMPUTE MATRICES A,B,C AND VECTOR R IN FINITE DIFFERENCE FORM
C
C      CALL ABCRN(A,B,C,I,N,IR,IC,DX)
C
C      COMPUTE ALPHA AND BETA MATRICES
C
C      CALL ALPBET(A,B,C,R,BET,FNC1,I,N,IR,IC,DETERM)
      LET=CABS(DETERM)
      IF(DET) 7,99,7
7      CONTINUE
      DO 4 I=1,IR
      FNC(I)=BET(I)
4      FNC1(I)=BET(I)
10 PRINT 100,FNC
      DO 5 I=2,N
      BACKSPACE 4
      BACKSPACE 4
      READ(4) A,BET
C
C      COMPUTE FUNCTION
C
C      CALL SOL(A,BET,FNC,FNC1,IR,IC)
11 PRINT 100,FNC
      DO 3 J=1,IR
      FNC1(J)=FNC(J)
      GO TO 97
99 PRINT 100,DETERM
97 CONTINUE
      STOP
      END

```



```

C      SUBROUTINE ABCR
C
C      PURPOSE
C      USER-PROVIDED SUBROUTINE THAT SETS UP EQUATIONS TO BE
C      SOLVED. LOWER BOUNDARY CONDITIONS ARE GIVEN IN STATEMENT 10
C      TO 11, UPPER BOUNDARY CONDITIONS IN 4 TO 12, AND EQUATIONS
C      IN 5 TO 3.
C
C      USAGE
C      SET UP MATRICES A,B,C AND VECTOR R.
C
C      DESCRIPTION OF PARAMETERS
C      A1 - OUTPUT MATRIX
C      B1 - OUTPUT MATRIX
C      C1 - OUTPUT MATRIX
C      R1 - OUTPUT VECTOR
C
C      REMARKS
C      USER-PROVIDED. THE SOLUTION TO THIS EXAMPLE IS:
C       $F(X) = (X^{**2}, I*(X^{**3}), X^{**4} + 3*X*I)$ 
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      NONE
C
C      METHOD
C      USER SUPPLIED

```

```

C .....
C
C      SUBROUTINE ABCR(A1,B1,C1,R1,K,N,X,IR,IC)
C
C      CHANGE DIMENSIONS
C
C      COMPLEX A1(3,3),B1(3,3),C1(3,3),R1(3)
C      DO 1 I=1,IR
C      DO 1 J=1,IC
1   A1(I,J)=B1(I,J)=C1(I,J)=CMPLX(0.0,0.0)
10  IF(K-1) 2,2,3
2   C1(1,1)=C1(2,3)=CMPLX(1.,0.0)
   A1(2,3)=CMPLX(2.,0.0)
   A1(3,2)=CMPLX(3.,0.0)
   R1(1)=R1(3)=CMPLX(0.0,0.0)
11  R1(2)=CMPLX(0.0,3.0)
   GO TO 6
3   IF(K-N) 5,4,4
4   A1(1,1)=CMPLX(1.,0.0)
   C1(3,2)=CMPLX(2.,0.0)
   A1(2,3)=CMPLX(1.,0.0)
   A=X*X
   A1(3,2)=CMPLX(A,0.0)
   E=A*A
   BI=3.*X
   C1(1)=X*X+A*A*X
   R1(1)=CMPLX(A,0.0)
   R1(2)=CMPLX(B,BI)
12  R1(3)=CMPLX(0.0,C1)
   GO TO 6

```

```

5 C1(1,1)=CMPLX(2.0,0.0)
  T1=X*X
  T2=X*T1
  T3=T2*X
  T4=T3*X
  T5=T4*X
  C1(2,1)=A1(1,3)=CMPLX(0.0,T2)
  C1(2,2)=A1(2,2)=A1(3,3)=B1(3,3)=CMPLX(T1,0.0)
  R=4.-3.*T2
  RI=4.*T5
  R1(1)=CMPLX(R,RI)
  RI=5.*T2+3.*T3
  R1(2)=CMPLX(0.0,RI)
  R=4.*T4+T5
  RI=3.*T1+3.*T2
  R1(3)=CMPLX(R,RI)
6 CONTINUE
  RETURN
  END

```

```

C      SUBROUTINE ABCRN
C
C      PURPOSE
C      CONVERTS DIFFERENTIAL FORM OF EQUATIONS TO FINITE DIFFER-
C      ENCE FORM.
C
C      USAGE
C      INPUT MATRICES A,B,C AND X VALUE. OUTPUT IN A,B,C.
C
C      DESCRIPTION OF PARAMETERS
C      A - INPUT,OUTPUT MATRIX
C      B - INPUT,OUTPUT MATRIX
C      K - STEP NUMBER
C      N - FINAL STEP NUMBER
C      IR - NUMBER OF ROWS
C      IC - NUMBER OF COLUMNS
C      DX - STEP SIZE
C
C      REMARKS
C      DIMENSIONS MUST BE CHANGED
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      NONE
C
C      METHOD
C      FINITE DIFFERENCE APPROXIMATION TO DIFFERENTIAL.
C.....
C
C      SUBROUTINE ABCRN(A,B,C,K,N,IR,IC,DX)
C
C      CHANGE DIMENSIONS
C
C      COMPLEX A(3,3),B(3,3),C(3,3)
C      F=1./DX
C      IF(K-1) 1,1,2
C      IF(K-N) 3,8,8
C      DO 10 I=1,IR
C      DO 10 J=1,IC
C      A(I,J)=A(I,J)-F*C(I,J)
C      E(I,J)=F*C(I,J)
C      GO TO 6
C      3      F2=F*F
C      F3=F/2.
C      F4=-2.*F2
C      DO 20 I=1,IR
C      DO 20 J=1,IC
C      B(I,J)=3(I,J)+F4*C(I,J)
C      C(I,J)=F2*C(I,J)+F3*A(I,J)
C      20      A(I,J)=C(I,J)-F*A(I,J)
C      GO TO 6
C      3      DO 30 I=1,IR
C      DO 30 J=1,IC
C      B(I,J)=A(I,J)+F*C(I,J)
C      30      A(I,J)=-F*C(I,J)
C      RETURN
C      END

```

```

C      SUBROUTINE ALPBET
C
C      PURPOSE
C          CALCULATES ALPHA AND BETA WHICH ARE NEEDED AS INTERMEDIATE
C          SOLUTIONS.  OUTPUT APPEARS ON TAPE4.
C
C      USAGE
C          PROVIDE INPUT MATRICES A,B,C AND VECTOR D.  RETURN ALPHA
C          AND BETA ON TAPE4.
C
C      DESCRIPTION OF PARAMETERS
C          A - INPUT MATRIX
C          B - INPUT MATRIX
C          C - INPUT MATRIX
C          D - INPUT VECTOR
C          BET - WORKING VECTOR
C          FNC1 - WORKING VECTOR
C          K - STEP NUMBER
C          N - FINAL STEP NUMBER
C          IR - NUMBER OF ROWS
C          IC - NUMBER COLUMNS
C          DETERM - VALUE OF DETERMINANT
C
C      REMARKS
C          TAPE3 IS A WORK TAPE.  IN MANY CASES IT CAN BE REPLACED
C          WITH MATRICES TO CUT RUN TIME
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          MINV - MATRIX INVERSION
C          MPRDD - MATRIX MULTIPLICATION
C          GMADD - MATRIX ADDITION
C
C      METHOD
C          SEE REFERENCE
C
C.....
C
C      SUBROUTINE ALPBET(A,B,C,D,BET,FNC1,K,N,IR,IC,DETERM)
C
C      CHANGE DIMENSIONS
C
C      COMPLEX A(3,3),B(3,3),C(3,3),D(3),BET(3),FNC1(3),DETERM
100  FORMAT(6(1X,E13.6))
101  FORMAT(1X,I4)
      IF(K-1) 1,1,2
1      CALL MINV(A,IC,D,1,DETERM)
      DO 11 KI=1,IR
      DO 11 JI=1,IC
11     A(KI,JI)=-A(KI,JI)
      CALL MPRDD(A,B,C,IR,IC,IC)
C
C      WRITE ALPHA ZERO AND BETA ZERO ON TAPES 3 AND 4
C      TAPE3 IS A WORKING TAPE.  TAPE4 STORES ALL ALPHA N'S AND BETA N'S
C
      WRITE(3) C,D
      WRITE(4) C,D
      RETURN

```



```

2    CONTINUE
C
C    STORE CN AND BN ON TAPES
C
    WRITE(3) C,B
    REWIND 3
C
C    READ AL(N-1) AND BET(N-1) FROM TAPES
C
    READ(3) C,BET
C
    CALL MPRDD(A,BET,FNC1,IR,IC,1)
    CALL GMADD(D,FNC1,BET,IR,1,-1.)
    CALL MPRDD(A,C,B,IR,IC,IC)
    DO 4 I=1,IR
    DO 4 J=1,IC
4    A(I,J)=B(I,J)
C
C    RETRIEVE CN AND BN FROM TAPES
C
    READ(3) C,B
    DO 5 I=1,IR
    DO 5 J=1,IC
5    C(I,J)=-C(I,J)
    REWIND 3
C
    CALL GMADD(B,A,B,IR,IC,1.)
    CALL MINV(B,IC,BET,1,DETERM)
    CALL MPRDD(B,C,A,IR,IC,IC)
C
C    WRITE BET(N) AND AL(N) ON TAPES
C
    WRITE(3) A,BET
C
C    WRITE BET(N) AND AL(N) ON TAPE4
C
    WRITE(4) A,BET
    RETURN
    END

```

SUBROUTINE SOL

PURPOSE

GIVEN INTERMEDIATE SOLUTIONS ALPHA AND BETA AND PREVIOUS
VALUE OF $F(X)$, SOLVES FOR $F(X-DX)$.

USAGE

PROVIDE ALPHA, BETA, AND $F(X)$. RETURNS $F(X-DX)$.

DESCRIPTION OF PARAMETERS

ALP - INPUT MATRIX ALPHA

BET - INPUT VECTOR BETA

FNC1 - PREVIOUS VECTOR SOLUTION $F(X)$

FNC - RETURNED VECTOR SOLUTION $F(X-DX)$

REMARKS

NONE

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

GMADD - MATRIX ADDITION

MPROD - MATRIX MULTIPLICATION

METHOD

SEE REFERENCE.

.....
SUBROUTINE SOL (ALP,BET,FNC,FNC1,IR,IC)

(CHANGE DIMENSIONS

COMPLEX ALP(3,3), BET(3), FNC(3), FNC1(3)

CALL MPROD(ALP,FNC1,FNC,IR,IC,1)

CALL GMADD(FNC,BET,FNC,IR,1,1.)

RETURN

END

```

C      SUBROUTINE MINV
C
C      PURPOSE
C      MATRIX INVERSION WITH ACCOMPANING SOLUTIONS OF LINEAR
C      EQUATIONS.
C
C      USAGE
C      INPUT MATRICES A AND B. RETURNS A-INVERSE IN A AND A-INVERSE
C      TIMES B IN B.
C
C      DESCRIPTION OF PARAMETERS
C      A - INPUT MATRIX TO BE INVERTED
C      INVERSE RETURNED IN A.
C      B - MATRIX OR VECTOR SUCH THAT  $AX=B$ .
C      A-INVERSE TIMES B RETURNED IN B.
C      N - COLUMNS IN A, ROWS IN B.
C      M - COLUMNS IN B.
C      DETERM - DETERMINANT OF A
C
C      REMARKS
C      F1 NBSB MATINV MATRIX INVERSION
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      CABS - SYSTEM FUNCTION
C
C      METHOD
C      GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.
C
C      .....
C
C      SUBROUTINE MINV(A,N,B,M,DETERM)
C
C      CHANGE DIMENSIONS
C
C      DIMENSION A(3,3),B(3,1),PIVOT(3),IPIVOT(3),INDEX(3,2)
C      COMPLEX      A,B,DETERM,PIVOT,AMAX,SWAP,T
C
C      INITIALIZATION
C
10     DETERM= CMPLX (1.0,0.0)
15     DO 20 J=1,N
20     IPIVOT(J)=0
30     DO 50 I=1,N
C
C      SEARCH FOR PIVOT ELEMENT
C
40     AMAX= CMPLX (0.0,0.0)
45     DO 105 J=1,N
50     IF(IPIVOT(J)-1)60,105,60
60     DO 100 K=1,N
70     IF(IPIVOT(K)-1)80,100,740
80     IF(CABS(AMAX)-CABS(A(J,K)))85,100,100
85     IPON=J
90     ICOLUM=K
95     AMAX=A(J,K)
100    CONTINUE
105    CONTINUE

```

```

110  IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C  INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
130  IF(IROW-ICOLUM) 1,0,250,140
140  [ETERM=-DETERM
150  DO 210 L=1,N
160  SWAP=A(IROW,L)
170  A(IROW,L)=A(ICOLUM,L)
200  A(ICOLUM,L)=SWAP
205  IF(M)260,260,210
210  DO 250 L=1,M
220  SWAP=B(IROW,L)
230  B(IROW,L)=B(ICOLUM,L)
250  B(ICOLUM,L)=SWAP
260  INDEX(I,1)=IROW
270  INDEX(I,2)=ICOLUM
310  PIVOT(I)=A(ICOLUM,ICOLUM)
320  DETERM=DETERM*PIVOT(I)
    LET=CABS(DETERM)
    IF(DET  )330,740,330
C
C  DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
330  A(ICOLUM,ICOLUM)= CMPLX(1.0,0.0)
340  DO 350 L=1,N
350  A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
355  IF(M)380,380,360
360  DO 370 L=1,M
370  B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C
C  REDUCE NON-PIVOT ROWS
C
380  DO 550 L1=1,N
390  IF(L1-ICOLUM)400,550,400
400  T=A(L1,ICOLUM)
420  A(L1,ICOLUM)= CMPLX(0.0,0.0)
430  DO 450 L=1,N
450  A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
455  IF(M)550,550,-60
460  DO 500 L=1,M
500  B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
550  CONTINUE
C
C  INTERCHANGE COLUMNS
C
600  DO 710 I=1,N
610  L=N+1-I
620  IF(INDEX(L,1)-INDEX(L,2))630,710,630
630  JROW=INDEX(L,1)
640  JCOLUM=INDEX(L,2)
650  DO 700 K=1,N
660  SWAP=A(K,JROW)
670  A(K,JROW)=A(K,JCOLUM)
700  A(K,JCOLUM)=SWAP
705  CONTINUE
710  CONTINUE
740  RETURN
    END

```



```

C      SUBROUTINE GMADD
C
C      PURPOSE
C      ADD TWO GENERAL MATRICES TO FORM RESULTANT GENERAL MATRIX
C
C      USAGE
C      CALL GMADD(A,B,R,N,M,AOS)
C
C      DESCRIPTION OF PARAMETERS
C      A - NAME OF FIRST INPUT MATRIX
C      B - NAME OF SECOND INPUT MATRIX
C      R - NAME OF OUTPUT MATRIX
C      N - NUMBER OF ROWS IN A,B,R
C      M - NUMBER OF COLUMNS IN A,B,R
C
C      REMARKS
C      ALL MATRICES MUST BE STORED AS GENERAL MATRICES
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      NONE
C
C      METHOD
C      ADDITION IS PERFORMED ELEMENT BY ELEMENT
C
C      .....
C
C      SUBROUTINE GMADD(A,B,R,N,M,AOS)
C      COMPLEX A(1),B(1),R(1)
C
C      CALCULATE NUMBER OF ELEMENTS
C
C      NM=N*M
C
C      ADD MATRICES
C
C      DO 10 I=1,NM
10    R(I)=A(I)+AOS*B(I)
C      RETURN
C      END

```

```

C      SUBROUTINE MPRDD
C
C      PURPOSE
C      MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL
C      MATRIX
C
C      USAGE
C      CALL MPRDD(A,B,R,N,M,L)
C
C      DESCRIPTION OF PARAMETERS
C      A - NAME OF FIRST INPUT MATRIX
C      B - NAME OF SECOND INPUT MATRIX
C      R - NAME OF OUTPUT MATRIX
C      N - NUMBER OF ROWS IN A
C      M - NUMBER OF COLUMNS IN A AND ROWS IN B
C      L - NUMBER OF COLUMNS IN B
C
C      REMARKS
C      ALL MATRICES MUST BE STORED AS GENERAL MATRICES
C      MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A
C      MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX B
C      NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROW
C      OF MATRIX B
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C
C      NONE
C
C      METHOD
C      THE M BY L MATRIX B IS PREMULIPLIED BY THE N BY M MATRIX A
C      AND THE RESULT IS STORED IN THE N BY L MATRIX R.
C
C      .....
C
C      SUBROUTINE MPRDD(A,B,R,N,M,L)
C      COMPLEX A(1),B(1),R(1)
C
C      IR=0
C      IK=-M
C      DO 10 K=1,L
C      IK=IK+M
C      DO 10 J=1,N
C      IR=IR+1
C      JI=J-N
C      IB=IK
C      F(IR)=(0.0,0.0)
C      DO 10 I=1,M
C      JI=JI+N
C      IB=IB+1
C      10 F(IR)=R(IR)+A(JI)*B(IB)
C      RETURN
C      END

```

APPENDIX II

$x^4 + 3x^1$

$1x^3$

x^2

$x^0 - n\delta x = x$

.200000E+01	.400000E+01	0.	.000000E+01	.160000E+02	.600000E+01
.199000E+01	.396010E+01	0.	.788060E+01	.156024E+02	.597000E+01
.198000E+01	.392040E+01	0.	.782339E+01	.153635E+02	.59-000E+01
.197000E+01	.388090E+01	0.	.764537E+01	.150614E+02	.591000E+01
.196000E+01	.384160E+01	0.	.752354E+01	.147579E+02	.586000E+01
.195000E+01	.380250E+01	0.	.741427E+01	.144390E+02	.583000E+01
.194000E+01	.376360E+01	0.	.730138E+01	.141647E+02	.582000E+01
.193000E+01	.372490E+01	0.	.718306E+01	.138749E+02	.579000E+01
.192000E+01	.368640E+01	0.	.707139E+01	.135835E+02	.576000E+01
.191000E+01	.364810E+01	0.	.696737E+01	.133036E+02	.573000E+01
.190000E+01	.361000E+01	0.	.685300E+01	.130321E+02	.570000E+01
.189000E+01	.357210E+01	0.	.675127E+01	.127599E+02	.567000E+01
.188000E+01	.353440E+01	0.	.664467E+01	.124920E+02	.564000E+01
.187000E+01	.349690E+01	0.	.653920E+01	.122283E+02	.561000E+01
.186000E+01	.345950E+01	0.	.643436E+01	.119686E+02	.558000E+01
.185000E+01	.342250E+01	0.	.633162E+01	.117135E+02	.555000E+01
.184000E+01	.338560E+01	0.	.622950E+01	.114623E+02	.552000E+01
.183000E+01	.334890E+01	0.	.612849E+01	.112151E+02	.549000E+01
.182000E+01	.331240E+01	0.	.602357E+01	.109720E+02	.546000E+01
.181000E+01	.327610E+01	0.	.592374E+01	.107329E+02	.543000E+01
.180000E+01	.324000E+01	0.	.583200E+01	.104976E+02	.540000E+01
.179000E+01	.320410E+01	0.	.573534E+01	.102663E+02	.537000E+01
.178000E+01	.316840E+01	0.	.563975E+01	.100388E+02	.534000E+01
.177000E+01	.313290E+01	0.	.554523E+01	.981506E+01	.531000E+01
.176000E+01	.309760E+01	0.	.545173E+01	.959135E+01	.528000E+01
.175000E+01	.306250E+01	0.	.535937E+01	.937891E+01	.525000E+01
.174000E+01	.302760E+01	0.	.526802E+01	.916636E+01	.522000E+01
.173000E+01	.299290E+01	0.	.517772E+01	.895705E+01	.519000E+01
.172000E+01	.295840E+01	0.	.508845E+01	.875213E+01	.516000E+01
.171000E+01	.292410E+01	0.	.500021E+01	.855036E+01	.513000E+01
.170000E+01	.289000E+01	0.	.491300E+01	.835216E+01	.510000E+01
.169000E+01	.285610E+01	0.	.482581E+01	.815731E+01	.507000E+01
.168000E+01	.282240E+01	0.	.474163E+01	.796574E+01	.504000E+01
.167000E+01	.278890E+01	0.	.465746E+01	.777736E+01	.501000E+01
.166000E+01	.275560E+01	0.	.457430E+01	.759333E+01	.498000E+01
.165000E+01	.272250E+01	0.	.449212E+01	.741201E+01	.495000E+01
.164000E+01	.268960E+01	0.	.441094E+01	.723395E+01	.492000E+01
.163000E+01	.265690E+01	0.	.433075E+01	.705912E+01	.489000E+01
.162000E+01	.262440E+01	0.	.425153E+01	.688705E+01	.486000E+01
.161000E+01	.259210E+01	0.	.417323E+01	.671898E+01	.483000E+01
.160000E+01	.256000E+01	0.	.409500E+01	.655350E+01	.480000E+01
.159000E+01	.252810E+01	0.	.401663E+01	.639123E+01	.477000E+01

.158000E+01	.246640E+01	0.	.394431E+01	.623201E+01	.474000E+01
.157000E+01	.24630E+01	0.	.335939E+01	.607573E+01	.471000E+01
.156000E+01	.245350E+01	0.	.374642E+01	.592241E+01	.468000E+01
.155000E+01	.240250E+01	0.	.372337E+01	.577201E+01	.465000E+01
.154000E+01	.237150E+01	0.	.367225E+01	.562--9E+01	.462000E+01
.153000E+01	.234030E+01	0.	.356153E+01	.547581E+01	.459000E+01
.152000E+01	.231240E+01	0.	.351181E+01	.533795E+01	.456000E+01
.151000E+01	.228010E+01	0.	.344295E+01	.519686E+01	.453000E+01
.150000E+01	.225000E+01	0.	.337500E+01	.506250E+01	.450000E+01
.149000E+01	.222010E+01	0.	.330735E+01	.492884E+01	.447000E+01
.148000E+01	.219040E+01	0.	.324173E+01	.479785E+01	.444000E+01
.147000E+01	.216030E+01	0.	.317652E+01	.466549E+01	.441000E+01
.146000E+01	.213120E+01	0.	.311214E+01	.454372E+01	.438000E+01
.145000E+01	.210250E+01	0.	.304362E+01	.442051E+01	.435000E+01
.144000E+01	.207360E+01	0.	.296593E+01	.429932E+01	.432000E+01
.143000E+01	.204430E+01	0.	.292421E+01	.418152E+01	.429000E+01
.142000E+01	.201640E+01	0.	.286323E+01	.406537E+01	.426000E+01
.141000E+01	.198810E+01	0.	.280322E+01	.395254E+01	.423000E+01
.140000E+01	.196000E+01	0.	.274400E+01	.384150E+01	.420000E+01
.139000E+01	.193210E+01	0.	.268562E+01	.373601E+01	.417000E+01
.138000E+01	.190440E+01	0.	.262307E+01	.362674E+01	.414000E+01
.137000E+01	.187630E+01	0.	.257135E+01	.352275E+01	.411000E+01
.136000E+01	.184860E+01	0.	.251545E+01	.342102E+01	.408000E+01
.135000E+01	.182250E+01	0.	.246037E+01	.332151E+01	.405000E+01
.134000E+01	.179560E+01	0.	.240610E+01	.322418E+01	.402000E+01
.133000E+01	.176890E+01	0.	.235250E+01	.312901E+01	.399000E+01
.132000E+01	.174240E+01	0.	.229977E+01	.303536E+01	.396000E+01
.131000E+01	.171610E+01	0.	.224809E+01	.294509E+01	.393000E+01
.130000E+01	.169000E+01	0.	.219700E+01	.285810E+01	.390000E+01
.129000E+01	.166410E+01	0.	.214669E+01	.276923E+01	.387000E+01
.128000E+01	.163840E+01	0.	.209715E+01	.268438E+01	.384000E+01
.127000E+01	.161230E+01	0.	.204833E+01	.260145E+01	.381000E+01
.126000E+01	.158750E+01	0.	.200038E+01	.252047E+01	.378000E+01
.125000E+01	.156250E+01	0.	.195312E+01	.244141E+01	.375000E+01
.124000E+01	.153750E+01	0.	.190562E+01	.236421E+01	.372000E+01
.123000E+01	.151290E+01	0.	.186087E+01	.228697E+01	.369000E+01
.122000E+01	.148840E+01	0.	.181535E+01	.221573E+01	.366000E+01
.121000E+01	.146410E+01	0.	.177155E+01	.214555E+01	.363000E+01
.120000E+01	.144000E+01	0.	.172800E+01	.207550E+01	.360000E+01
.119000E+01	.141610E+01	0.	.168516E+01	.200534E+01	.357000E+01
.118000E+01	.139240E+01	0.	.164303E+01	.193795E+01	.354000E+01
.117000E+01	.136830E+01	0.	.160161E+01	.187399E+01	.351000E+01

.116000E+01	.134560E+01	0.	.156030E+01	.181054E+01	.348000E+01
.115000E+01	.132250E+01	0.	.152097E+01	.174901E+01	.345000E+01
.114000E+01	.129960E+01	0.	.149154E+01	.166936E+01	.342000E+01
.113000E+01	.127690E+01	0.	.144290E+01	.163047E+01	.339000E+01
.112000E+01	.125440E+01	0.	.140433E+01	.157352E+01	.336000E+01
.111000E+01	.123210E+01	0.	.136733E+01	.151607E+01	.333000E+01
.110000E+01	.121000E+01	0.	.133100E+01	.146410E+01	.330000E+01
.109000E+01	.118810E+01	0.	.129503E+01	.141158E+01	.327000E+01
.108000E+01	.116640E+01	0.	.125371E+01	.136049E+01	.324000E+01
.107000E+01	.114430E+01	0.	.122504E+01	.131050E+01	.321000E+01
.106000E+01	.112360E+01	0.	.119102E+01	.126248E+01	.318000E+01
.105000E+01	.110260E+01	0.	.11732E+01	.121551E+01	.315000E+01
.104000E+01	.108160E+01	0.	.112436E+01	.116986E+01	.312000E+01
.103000E+01	.106030E+01	0.	.109273E+01	.112551E+01	.309000E+01
.102000E+01	.104040E+01	0.	.106121E+01	.108243E+01	.306000E+01
.101000E+01	.102010E+01	0.	.103030E+01	.104050E+01	.303000E+01
.100000E+01	.100000E+01	0.	.100000E+01	.100000E+01	.300000E+01
.990000E+00	.980100E+00	0.	.970299E+00	.960596E+00	.297000E+01
.980000E+00	.960400E+00	0.	.941132E+00	.922358E+00	.294000E+01
.970000E+00	.940900E+00	0.	.912673E+00	.885235E+00	.291000E+01
.960000E+00	.921600E+00	0.	.884735E+00	.849347E+00	.288000E+01
.950000E+00	.902500E+00	0.	.857375E+00	.814065E+00	.285000E+01
.940000E+00	.883600E+00	0.	.830534E+00	.780743E+00	.282000E+01
.930000E+00	.864900E+00	0.	.804357E+00	.748052E+00	.279000E+01
.920000E+00	.846400E+00	0.	.778586E+00	.718333E+00	.276000E+01
.910000E+00	.828100E+00	0.	.753571E+00	.685750E+00	.273000E+01
.900000E+00	.810000E+00	0.	.729000E+00	.656100E+00	.270000E+01
.890000E+00	.792100E+00	0.	.704963E+00	.627422E+00	.267000E+01
.880000E+00	.774400E+00	0.	.681472E+00	.599635E+00	.264000E+01
.870000E+00	.756900E+00	0.	.658503E+00	.572596E+00	.261000E+01
.860000E+00	.739600E+00	0.	.636056E+00	.547008E+00	.258000E+01
.850000E+00	.722500E+00	0.	.614125E+00	.522006E+00	.255000E+01
.840000E+00	.705400E+00	0.	.592704E+00	.497671E+00	.252000E+01
.830000E+00	.688500E+00	0.	.571787E+00	.474583E+00	.249000E+01
.820000E+00	.672400E+00	0.	.551363E+00	.452122E+00	.246000E+01
.810000E+00	.656100E+00	0.	.531441E+00	.430457E+00	.243000E+01
.800000E+00	.640000E+00	0.	.512000E+00	.409600E+00	.240000E+01
.790000E+00	.624100E+00	0.	.493033E+00	.391010E+00	.237000E+01
.780000E+00	.608400E+00	0.	.475525E+00	.370151E+00	.234000E+01
.770000E+00	.592500E+00	0.	.458533E+00	.351530E+00	.231000E+01
.760000E+00	.577600E+00	0.	.443750E+00	.334222E+00	.228000E+01
.750000E+00	.562160E+00	0.	.421375E+00	.316406E+00	.225000E+01

.740000E+00	.547000E+00	0.	.605224E+00	.293055E+00	.222000E+01
.730000E+00	.532900E+00	0.	.389017E+00	.283992E+00	.219000E+01
.720000E+00	.518400E+00	0.	.373248E+00	.266739E+00	.216000E+01
.710000E+00	.504100E+00	0.	.357911E+00	.254117E+00	.213000E+01
.700000E+00	.490000E+00	0.	.343000E+00	.240100E+00	.210000E+01
.690000E+00	.476100E+00	0.	.325039E+00	.226671E+00	.207000E+01
.680000E+00	.462400E+00	0.	.314432E+00	.213814E+00	.204000E+01
.670000E+00	.448900E+00	0.	.300763E+00	.201511E+00	.201000E+01
.660000E+00	.435600E+00	0.	.287435E+00	.189747E+00	.198000E+01
.650000E+00	.422500E+00	0.	.274025E+00	.178505E+00	.195000E+01
.640000E+00	.409600E+00	0.	.262144E+00	.167772E+00	.192000E+01
.630000E+00	.396900E+00	0.	.250047E+00	.157530E+00	.189000E+01
.620000E+00	.384400E+00	0.	.238328E+00	.147763E+00	.186000E+01
.610000E+00	.372100E+00	0.	.226981E+00	.136458E+00	.183000E+01
.600000E+00	.360000E+00	0.	.216000E+00	.129600E+00	.180000E+01
.590000E+00	.348100E+00	0.	.205379E+00	.121174E+00	.177000E+01
.580000E+00	.336400E+00	0.	.195112E+00	.113165E+00	.174000E+01
.570000E+00	.324900E+00	0.	.185193E+00	.105560E+00	.171000E+01
.560000E+00	.313600E+00	0.	.175015E+00	.983450E-01	.168000E+01
.550000E+00	.302500E+00	0.	.166375E+00	.915052E-01	.165000E+01
.540000E+00	.291600E+00	0.	.157405E+00	.850305E-01	.162000E+01
.530000E+00	.280900E+00	0.	.148877E+00	.789048E-01	.159000E+01
.520000E+00	.270400E+00	0.	.140508E+00	.731162E-01	.156000E+01
.510000E+00	.260100E+00	0.	.132651E+00	.676520E-01	.153000E+01
.500000E+00	.250000E+00	0.	.125000E+00	.625000E-01	.150000E+01
.490000E+00	.240100E+00	0.	.117049E+00	.576480E-01	.147000E+01
.480000E+00	.230400E+00	0.	.110592E+00	.530842E-01	.144000E+01
.470000E+00	.220900E+00	0.	.103323E+00	.487558E-01	.141000E+01
.460000E+00	.211600E+00	0.	.973360E-01	.447745E-01	.138000E+01
.450000E+00	.202500E+00	0.	.911250E-01	.410652E-01	.135000E+01
.440000E+00	.193600E+00	0.	.851840E-01	.374810E-01	.132000E+01
.430000E+00	.184900E+00	0.	.795070E-01	.341889E-01	.129000E+01
.420000E+00	.176400E+00	0.	.740330E-01	.311170E-01	.126000E+01
.410000E+00	.168100E+00	0.	.689210E-01	.282576E-01	.123000E+01
.400000E+00	.160000E+00	0.	.640000E-01	.256000E-01	.120000E+01
.390000E+00	.152100E+00	0.	.593190E-01	.231344E-01	.117000E+01
.380000E+00	.144400E+00	0.	.548720E-01	.208514E-01	.114000E+01
.370000E+00	.136900E+00	0.	.506530E-01	.187416E-01	.111000E+01
.360000E+00	.129600E+00	0.	.466560E-01	.167962E-01	.108000E+01
.350000E+00	.122500E+00	0.	.429750E-01	.150052E-01	.105000E+01
.340000E+00	.115600E+00	0.	.393040E-01	.133634E-01	.102000E+01
.330000E+00	.108900E+00	0.	.359370E-01	.118532E-01	.990000E+00

.320000E+00	.162000E+00	0.	.327000E-01	.104658E-01	.960000E+00
.310000E+00	.961000E-01	0.	.297310E-01	.523521E-02	.930000E+00
.300000E+00	.900000E-01	0.	.270000E-01	.610000E-02	.900000E+00
.290000E+00	.841000E-01	0.	.243830E-01	.707281E-02	.870000E+00
.280000E+00	.784000E-01	0.	.219520E-01	.614656E-02	.840000E+00
.270000E+00	.729000E-01	0.	.196830E-01	.531441E-02	.810000E+00
.260000E+00	.676000E-01	0.	.175760E-01	.456976E-02	.780000E+00
.250000E+00	.625000E-01	0.	.150250E-01	.390625E-02	.750000E+00
.240000E+00	.576000E-01	0.	.130240E-01	.331776E-02	.720000E+00
.230000E+00	.529000E-01	0.	.121570E-01	.279841E-02	.690000E+00
.220000E+00	.484000E-01	0.	.100480E-01	.234256E-02	.660000E+00
.210000E+00	.441000E-01	0.	.926100E-02	.194431E-02	.630000E+00
.200000E+00	.400000E-01	0.	.800000E-02	.160000E-02	.600000E+00
.190000E+00	.361000E-01	0.	.683000E-02	.130321E-02	.570000E+00
.180000E+00	.324000E-01	0.	.583200E-02	.104976E-02	.540000E+00
.170000E+00	.289000E-01	0.	.491300E-02	.835210E-03	.510000E+00
.160000E+00	.256000E-01	0.	.409500E-02	.653550E-03	.480000E+00
.150000E+00	.225000E-01	0.	.337500E-02	.506250E-03	.450000E+00
.140000E+00	.196000E-01	0.	.274300E-02	.384160E-03	.420000E+00
.130000E+00	.169000E-01	0.	.215700E-02	.285610E-03	.390000E+00
.120000E+00	.144000E-01	0.	.172300E-02	.207350E-03	.360000E+00
.110000E+00	.121000E-01	0.	.133100E-02	.146410E-03	.330000E+00
.100000E+00	.100000E-01	0.	.100000E-02	.100000E-03	.300000E+00
.900000E-01	.810000E-02	0.	.729000E-03	.656100E-04	.270000E+00
.800000E-01	.640000E-02	0.	.512000E-03	.409600E-04	.240000E+00
.700000E-01	.490000E-02	0.	.343000E-03	.240100E-04	.210000E+00
.600000E-01	.360000E-02	0.	.216000E-03	.129600E-04	.180000E+00
.500000E-01	.250000E-02	0.	.120000E-03	.625000E-05	.150000E+00
.400000E-01	.160000E-02	0.	.640000E-04	.256000E-05	.120000E+00
.300000E-01	.900000E-03	0.	.270000E-04	.810000E-05	.900000E-01
.200000E-01	.400000E-03	0.	.800000E-05	.160000E-05	.600000E-01
.100000E-01	.100000E-03	0.	.100000E-05	.100000E-07	.300000E-01
0.	0.	0.	0.	0.	0.

APPENDIX III

$x^4 + 3x^3$ $1x^3$ x^2

.400000E+01	1.	-1.040048E-03	.602780E+01	.160000E+02	.500000E+01
.396020E+01	-1.475093E-05	-1.06129E-03	.790835E+01	.156322E+02	.597000E+01
.392060E+01	-9.50604E-05	-1.08379E-03	.779010E+01	.153695E+02	.594000E+01
.388120E+01	-1.44264E-04	-1.10643E-03	.767304E+01	.150612E+02	.591000E+01
.384200E+01	-1.07226E-04	-1.13059E-03	.755716E+01	.147579E+02	.588000E+01
.380300E+01	-2.42306E-04	-1.15482E-03	.744245E+01	.144589E+02	.585000E+01
.376420E+01	-2.91544E-04	-1.18042E-03	.732892E+01	.141647E+02	.582000E+01
.372560E+01	-3.40857E-04	-1.20602E-03	.721654E+01	.138747E+02	.579000E+01
.368720E+01	-3.90672E-04	-1.23285E-03	.710533E+01	.135896E+02	.576000E+01
.364900E+01	-4.40312E-04	-1.25961E-03	.699527E+01	.133085E+02	.573000E+01
.361100E+01	-4.90382E-04	-1.28746E-03	.688635E+01	.130322E+02	.570000E+01
.357320E+01	-5.40237E-04	-1.31519E-03	.677857E+01	.127598E+02	.567000E+01
.353560E+01	-5.90454E-04	-1.34388E-03	.667192E+01	.124921E+02	.564000E+01
.349820E+01	-6.40420E-04	-1.37273E-03	.656641E+01	.122282E+02	.561000E+01
.346100E+01	-6.90682E-04	-1.40173E-03	.646201E+01	.119689E+02	.558000E+01
.342400E+01	-7.40650E-04	-1.43083E-03	.635873E+01	.117134E+02	.555000E+01
.338720E+01	-7.90868E-04	-1.46006E-03	.625656E+01	.114624E+02	.552000E+01
.335060E+01	-8.40761E-04	-1.49022E-03	.615540E+01	.112151E+02	.549000E+01
.331420E+01	-8.90828E-04	-1.52036E-03	.605525E+01	.109721E+02	.546000E+01
.327800E+01	-9.40547E-04	-1.55022E-03	.595664E+01	.107323E+02	.543000E+01
.324200E+01	-9.90386E-04	-1.58085E-03	.585895E+01	.104977E+02	.540000E+01
.320620E+01	-1.03985E-03	-1.61154E-03	.576213E+01	.102662E+02	.537000E+01
.317060E+01	-1.08938E-03	-1.64293E-03	.566640E+01	.100389E+02	.534000E+01
.313520E+01	-1.13850E-03	-1.67401E-03	.557192E+01	.981506E+01	.531000E+01
.310000E+01	-1.18764E-03	-1.70422E-03	.547841E+01	.959528E+01	.528000E+01
.306500E+01	-1.23636E-03	-1.73407E-03	.538595E+01	.937892E+01	.525000E+01
.303020E+01	-1.28504E-03	-1.76411E-03	.528455E+01	.916653E+01	.522000E+01
.299560E+01	-1.33328E-03	-1.79480E-03	.520419E+01	.895748E+01	.519000E+01
.296120E+01	-1.38143E-03	-1.82068E-03	.511406E+01	.875232E+01	.516000E+01
.292700E+01	-1.42912E-03	-1.84986E-03	.502656E+01	.855041E+01	.513000E+01
.289300E+01	-1.47660E-03	-1.87922E-03	.493929E+01	.835233E+01	.510000E+01
.285920E+01	-1.52377E-03	-1.90804E-03	.485305E+01	.815737E+01	.507000E+01
.282560E+01	-1.57070E-03	-1.93690E-03	.476701E+01	.796616E+01	.504000E+01
.279220E+01	-1.61712E-03	-1.96518E-03	.468358E+01	.777804E+01	.501000E+01
.275900E+01	-1.66335E-03	-1.99342E-03	.460035E+01	.759356E+01	.498000E+01
.272600E+01	-1.70905E-03	-2.02160E-03	.451612E+01	.741211E+01	.495000E+01
.269320E+01	-1.75453E-03	-2.04986E-03	.443688E+01	.723419E+01	.492000E+01
.266060E+01	-1.79946E-03	-2.07556E-03	.435662E+01	.705923E+01	.489000E+01
.262820E+01	-1.84415E-03	-2.10239E-03	.427734E+01	.688774E+01	.486000E+01
.259600E+01	-1.88892E-03	-2.12966E-03	.419903E+01	.671912E+01	.483000E+01
.256400E+01	-1.93213E-03	-2.15645E-03	.412168E+01	.655388E+01	.480000E+01
.253220E+01	-1.97541E-03	-2.18176E-03	.404535E+01	.639144E+01	.477000E+01

.250060E+01	-.201838E-03	-.220495E-03	.396866E+01	.623230E+01	.474000E+01
.246920E+01	-.206078E-03	-.222937E-03	.389538E+01	.607590E+01	.471000E+01
.243800E+01	-.210285E-03	-.225352E-03	.382184E+01	.592272E+01	.468000E+01
.240700E+01	-.214433E-03	-.227699E-03	.374923E+01	.577220E+01	.465000E+01
.237620E+01	-.218545E-03	-.230014E-03	.367755E+01	.562481E+01	.462000E+01
.234560E+01	-.222598E-03	-.232261E-03	.360679E+01	.549002E+01	.459000E+01
.231520E+01	-.226614E-03	-.234473E-03	.353695E+01	.533829E+01	.456000E+01
.228500E+01	-.230570E-03	-.236616E-03	.346803E+01	.519908E+01	.453000E+01
.225500E+01	-.234466E-03	-.238721E-03	.340001E+01	.506286E+01	.450000E+01
.222520E+01	-.238343E-03	-.240755E-03	.333288E+01	.492909E+01	.447000E+01
.219560E+01	-.242158E-03	-.242749E-03	.326665E+01	.479823E+01	.444000E+01
.216620E+01	-.245913E-03	-.244673E-03	.320131E+01	.466976E+01	.441000E+01
.213700E+01	-.249625E-03	-.246553E-03	.313685E+01	.454411E+01	.438000E+01
.210800E+01	-.253276E-03	-.248363E-03	.307326E+01	.442079E+01	.435000E+01
.207920E+01	-.256884E-03	-.250128E-03	.301055E+01	.430023E+01	.432000E+01
.205060E+01	-.260430E-03	-.251822E-03	.294870E+01	.418192E+01	.429000E+01
.202220E+01	-.263932E-03	-.253469E-03	.288770E+01	.406630E+01	.426000E+01
.199400E+01	-.267373E-03	-.255045E-03	.282755E+01	.395287E+01	.423000E+01
.196600E+01	-.270768E-03	-.256572E-03	.276826E+01	.384204E+01	.420000E+01
.193820E+01	-.274193E-03	-.258029E-03	.270980E+01	.373336E+01	.417000E+01
.191060E+01	-.277390E-03	-.259435E-03	.265217E+01	.362720E+01	.414000E+01
.188320E+01	-.280617E-03	-.260772E-03	.259537E+01	.352312E+01	.411000E+01
.185600E+01	-.283797E-03	-.262057E-03	.253939E+01	.342150E+01	.408000E+01
.182900E+01	-.286917E-03	-.263272E-03	.248423E+01	.332189E+01	.405000E+01
.180220E+01	-.289989E-03	-.264435E-03	.242987E+01	.322468E+01	.402000E+01
.177560E+01	-.293001E-03	-.265529E-03	.237632E+01	.312941E+01	.399000E+01
.174920E+01	-.295964E-03	-.266570E-03	.232357E+01	.303648E+01	.396000E+01
.172300E+01	-.298869E-03	-.267542E-03	.227160E+01	.294543E+01	.393000E+01
.169700E+01	-.301725E-03	-.268461E-03	.222043E+01	.285664E+01	.390000E+01
.167120E+01	-.304522E-03	-.269311E-03	.217003E+01	.276968E+01	.387000E+01
.164560E+01	-.307271E-03	-.270108E-03	.212040E+01	.268491E+01	.384000E+01
.162020E+01	-.309962E-03	-.270838E-03	.207154E+01	.260192E+01	.381000E+01
.159500E+01	-.312605E-03	-.271513E-03	.202345E+01	.252105E+01	.378000E+01
.157000E+01	-.315189E-03	-.272123E-03	.197610E+01	.244190E+01	.375000E+01
.154520E+01	-.317726E-03	-.272675E-03	.192951E+01	.236481E+01	.372000E+01
.152060E+01	-.320206E-03	-.273168E-03	.188366E+01	.228938E+01	.369000E+01
.149620E+01	-.322638E-03	-.273603E-03	.183855E+01	.221595E+01	.366000E+01
.147200E+01	-.325015E-03	-.273975E-03	.179416E+01	.214413E+01	.363000E+01
.144800E+01	-.327343E-03	-.274292E-03	.175051E+01	.207424E+01	.360000E+01
.142420E+01	-.329617E-03	-.274547E-03	.170757E+01	.200590E+01	.357000E+01
.140060E+01	-.331844E-03	-.274747E-03	.166534E+01	.193943E+01	.354000E+01
.137720E+01	-.334017E-03	-.274896E-03	.162353E+01	.187474E+01	.351000E+01

.135400E+01	..336143F-03	..274971E-03	.158301E+01	.181132E+01	.348000E+01
.133100E+01	..335216E-03	..274996E-03	.154269E+01	.174961E+01	.345000E+01
.130820E+01	..340243F-03	..274967E-03	.150345E+01	.168966E+01	.342000E+01
.128560E+01	..342219F-03	..274880E-03	.146470F+01	.163110E+01	.339000E+01
.126320E+01	..344149F-03	..274739E-03	.142663E+01	.157424E+01	.336000E+01
.124100E+01	..346029F-03	..274541E-03	.138923E+01	.151872E+01	.333000E+01
.121900E+01	..347864E-03	..274290E-03	.135249E+01	.146484E+01	.330000E+01
.119720E+01	..349650F-03	..273983E-03	.131641E+01	.141226E+01	.327000E+01
.117560E+01	..351392E-03	..273625E-03	.128099E+01	.136125E+01	.324000E+01
.115420E+01	..353086F-03	..273210E-03	.124621E+01	.131149E+01	.321000E+01
.113300E+01	..354737E-03	..272746E-03	.121207E+01	.126326E+01	.318000E+01
.111200E+01	..356341E-03	..272227E-03	.117857E+01	.121623E+01	.315000E+01
.109120E+01	..357904E-03	..271659E-03	.114570E+01	.117067E+01	.312000E+01
.107060E+01	..359421E-03	..271037E-03	.111344E+01	.112626E+01	.309000E+01
.105020E+01	..360897E-03	..270367E-03	.108161E+01	.108327E+01	.306000E+01
.103000E+01	..362329F-03	..269644E-03	.105079F+01	.104138E+01	.303000E+01
.101000E+01	..363721E-03	..268875E-03	.102037E+01	.100086E+01	.300000E+01
.990200E+00	..365070E-03	..268054E-03	.990549E+00	.961393E+00	.297000E+01
.970600E+00	..366381E-03	..267186E-03	.961322E+00	.923249E+00	.294000E+01
.951200E+00	..367650F-03	..266270E-03	.932681E+00	.886115E+00	.291000E+01
.932000E+00	..368882F-03	..265307E-03	.904622F+00	.850251E+00	.288000E+01
.913000E+00	..370073E-03	..264296E-03	.877137E+00	.815355E+00	.285000E+01
.894200E+00	..371228F-03	..263240F-03	.850222E+00	.781678E+00	.282000E+01
.875600E+00	..372345F-03	..262137E-03	.823868E+00	.748926E+00	.279000E+01
.857200E+00	..373427F-03	..260990E-03	.798072E+00	.717347E+00	.276000E+01
.839000E+00	..374471F-03	..259797E-03	.772826E+00	.686659E+00	.273000E+01
.821000E+00	..375481E-03	..258561E-03	.748125E+00	.657080E+00	.270000E+01
.803200E+00	..376456E-03	..257251E-03	.723963E+00	.623350E+00	.267000E+01
.785600E+00	..377398E-03	..255958E-03	.700333E+00	.600701E+00	.264000E+01
.768200E+00	..378306F-03	..254592E-03	.677230E+00	.573852E+00	.261000E+01
.751000E+00	..379182E-03	..253184E-03	.654646E+00	.540039E+00	.258000E+01
.734000E+00	..380026E-03	..251735E-03	.632580E+00	.522989E+00	.255000E+01
.717200E+00	..380839E-03	..250244E-03	.611021E+00	.498929E+00	.252000E+01
.700600E+00	..381621F-03	..248712E-03	.589965E+00	.475593E+00	.249000E+01
.684200E+00	..382374E-03	..247141E-03	.569405E+00	.453206E+00	.246000E+01
.668000E+00	..383097E-03	..245537E-03	.549336E+00	.431506E+00	.243000E+01
.652000E+00	..383793E-03	..243879E-03	.529751E+00	.410712E+00	.240000E+01
.636200E+00	..384460F-03	..242189F-03	.511645E+00	.391568E+00	.237000E+01
.620600E+00	..385101F-03	..240462E-03	.492011E+00	.371290E+00	.234000E+01
.605200E+00	..385715E-03	..238696E-03	.473844E+00	.352627E+00	.231000E+01
.590000E+00	..386303E-03	..236892E-03	.456137E+00	.334789E+00	.228000E+01
.575000E+00	..386867E-03	..235051E-03	.438885F+00	.317532E+00	.225000E+01

.560200E+00	-.387406E-03	-.233174E-03	.422081E+00	.301062E+00	.222000E+01
.545600E+00	-.387921E-03	-.231259E-03	.405720E+00	.285138E+00	.219000E+01
.531200E+00	-.388414E-03	-.229309E-03	.389795E+00	.269964E+00	.216000E+01
.517000E+00	-.388884E-03	-.227323E-03	.374301E+00	.255302E+00	.213000E+01
.503000E+00	-.389332E-03	-.225302E-03	.359231E+00	.241354E+00	.210000E+01
.489200E+00	-.389760E-03	-.223245E-03	.344579E+00	.227668E+00	.207000E+01
.475600E+00	-.390167E-03	-.221154E-03	.330340E+00	.215098E+00	.204000E+01
.462200E+00	-.390554E-03	-.219028E-03	.316507E+00	.202759E+00	.201000E+01
.449000E+00	-.390922E-03	-.216868E-03	.303074E+00	.191062E+00	.198000E+01
.436000E+00	-.391271E-03	-.214673E-03	.290036E+00	.179785E+00	.195000E+01
.423200E+00	-.391603E-03	-.212446E-03	.277386E+00	.169118E+00	.192000E+01
.410600E+00	-.391917E-03	-.210184E-03	.265119E+00	.158841E+00	.189000E+01
.398200E+00	-.392215E-03	-.207889E-03	.253227E+00	.149140E+00	.186000E+01
.386000E+00	-.392496E-03	-.205561E-03	.241736E+00	.139802E+00	.183000E+01
.374000E+00	-.392762E-03	-.203201E-03	.230550E+00	.131009E+00	.180000E+01
.362200E+00	-.393013E-03	-.200807E-03	.219751E+00	.122550E+00	.177000E+01
.350600E+00	-.393250E-03	-.198381E-03	.209305E+00	.114606E+00	.174000E+01
.339200E+00	-.393472E-03	-.195922E-03	.199204E+00	.106970E+00	.171000E+01
.328000E+00	-.393682E-03	-.193432E-03	.189444E+00	.998183E-01	.168000E+01
.317000E+00	-.393879E-03	-.190908E-03	.180018E+00	.929502E-01	.165000E+01
.306200E+00	-.394063E-03	-.188353E-03	.170921E+00	.865372E-01	.162000E+01
.295600E+00	-.394236E-03	-.185766E-03	.162145E+00	.803832E-01	.159000E+01
.285200E+00	-.394398E-03	-.183147E-03	.153686E+00	.746566E-01	.156000E+01
.275000E+00	-.394549E-03	-.180496E-03	.145536E+00	.691653E-01	.153000E+01
.265000E+00	-.394690E-03	-.177813E-03	.137691E+00	.640743E-01	.150000E+01
.255200E+00	-.394821E-03	-.175098E-03	.130143E+00	.591968E-01	.147000E+01
.245600E+00	-.394943E-03	-.172352E-03	.122888E+00	.546939E-01	.144000E+01
.236200E+00	-.395056E-03	-.169573E-03	.115919E+00	.503817E-01	.141000E+01
.227000E+00	-.395161E-03	-.166763E-03	.109229E+00	.464197E-01	.138000E+01
.218000E+00	-.395258E-03	-.163921E-03	.102814E+00	.426277E-01	.135000E+01
.209200E+00	-.395347E-03	-.161046E-03	.966662E-01	.391621E-01	.132000E+01
.200600E+00	-.395429E-03	-.158140E-03	.907806E-01	.358466E-01	.129000E+01
.192200E+00	-.395505E-03	-.155202E-03	.851509E-01	.328347E-01	.126000E+01
.184000E+00	-.395574E-03	-.152232E-03	.797711E-01	.293539E-01	.123000E+01
.176000E+00	-.395638E-03	-.149229E-03	.746351E-01	.273550E-01	.120000E+01
.168200E+00	-.395696E-03	-.146194E-03	.697370E-01	.248690E-01	.117000E+01
.160600E+00	-.395748E-03	-.143126E-03	.650706E-01	.226442E-01	.114000E+01
.153200E+00	-.395796E-03	-.140026E-03	.606301E-01	.205151E-01	.111000E+01
.146000E+00	-.395840E-03	-.136892E-03	.564093E-01	.186274E-01	.108000E+01
.139000E+00	-.395879E-03	-.133726E-03	.524023E-01	.168192E-01	.105000E+01
.132200E+00	-.395914E-03	-.130527E-03	.486030E-01	.152337E-01	.102000E+01
.125600E+00	-.395945E-03	-.127294E-03	.450053E-01	.137123E-01	.990000E+00

.119200E+00	-.395973E-03	-.124027E-03	.416034E-01	.123958E-01	.360000E+00
.113000E+00	-.395904E-03	-.120727E-03	.783911E-01	.111291E-01	.330000E+00
.107000E+00	-.396121E-03	-.117303E-03	.257624E-01	.100504E-01	.300000E+00
.101200E+00	-.396040E-03	-.114024E-03	.325113E-01	.900810E-02	.270000E+00
.956000E-01	-.396057E-03	-.110621E-03	.295310E-01	.137944E-02	.240000E+00
.902000E-01	-.396072E-03	-.107183E-03	.273178E-01	.729177E-02	.210000E+00
.850000E-01	-.396086E-03	-.103710E-03	.249634E-01	.660285E-02	.180000E+00
.800000E-01	-.396097E-03	-.100222E-03	.227625E-01	.592635E-02	.150000E+00
.752000E-01	-.396107E-03	-.966575E-04	.207090E-01	.533325E-02	.120000E+00
.706000E-01	-.396115E-03	-.930774E-04	.187971E-01	.486194E-02	.090000E+00
.662000E-01	-.396122E-03	-.894611E-04	.170205E-01	.446116E-02	.060000E+00
.620000E-01	-.396128E-03	-.858082E-04	.153733E-01	.405247E-02	.030000E+00
.580000E-01	-.396133E-03	-.821183E-04	.138495E-01	.376243E-02	.000000E+00
.542000E-01	-.396137E-03	-.783912E-04	.124431E-01	.345572E-02	.570000E+00
.506000E-01	-.396140E-03	-.746265E-04	.111430E-01	.325676E-02	.540000E+00
.472000E-01	-.396143E-03	-.708239E-04	.995818E-02	.303330E-02	.510000E+00
.440000E-01	-.396145E-03	-.669829E-04	.866763E-02	.290769E-02	.480000E+00
.410000E-01	-.396146E-03	-.631032E-04	.787032E-02	.275067E-02	.450000E+00
.382000E-01	-.396148E-03	-.591846E-04	.696022E-02	.268257E-02	.420000E+00
.356000E-01	-.396149E-03	-.552264E-04	.613130E-02	.257711E-02	.390000E+00
.332000E-01	-.396149E-03	-.512245E-04	.537755E-02	.255264E-02	.360000E+00
.310000E-01	-.396150E-03	-.471904E-04	.469292E-02	.248578E-02	.330000E+00
.290000E-01	-.396150E-03	-.431116E-04	.407139E-02	.249295E-02	.300000E+00
.272000E-01	-.396151E-03	-.389919E-04	.351693E-02	.245364E-02	.270000E+00
.256000E-01	-.396151E-03	-.348307E-04	.296352E-02	.243239E-02	.240000E+00
.242000E-01	-.396151E-03	-.306277E-04	.252511E-02	.246151E-02	.210000E+00
.230000E-01	-.396151E-03	-.263625E-04	.205568E-02	.250370E-02	.180000E+00
.220000E-01	-.396151E-03	-.220946E-04	.169921E-02	.249404E-02	.150000E+00
.212000E-01	-.396151E-03	-.177636E-04	.131965E-02	.254346E-02	.120000E+00
.206000E-01	-.396151E-03	-.133431E-04	.980982E-03	.253974E-02	.090000E-01
.202000E-01	-.396151E-03	-.97063E-05	.647173E-03	.253203E-02	.060000E-01
.200000E-01	-.396151E-03	-.450774E-05	.322189E-03	.259094E-02	.030000E-01
.200000E-01	-.396151E-03			.264382E-02	-.516152E-11